McGill

ECSE 512 – Digital Signal Processing I
Fall 2010

FINAL EXAMINATION

9:00 am – 12:00 pm, December 20, 2010

Duration: 180 minutes

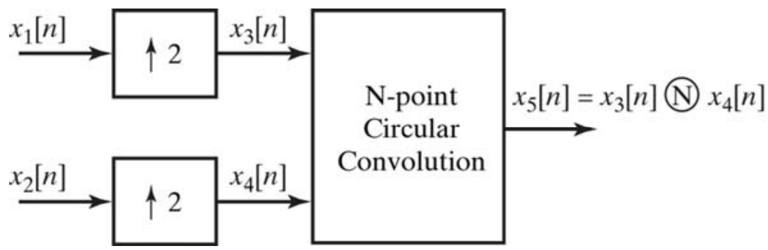
Examiner: Prof. M. Vu _______________ Assoc. Examiner: Prof. B. Champagne______________

- **There are 6 questions for a total of 120 points.**
- **This is a closed-book exam. You can bring 2 single-sided sheets of hand-written notes. These sheets of notes must be entirely hand-written, no portions may be machine-produced or photocopied.**
- **Calculators are permitted, but no cell phones or laptops are allowed.**
- **Attempt all questions.**

Question 1 (20 points) DFT

In the system shown in the figure below, $x_1[n]$ and $x_2[n]$ are both causal, 32-point sequences (that is, they are both zero outside the interval $0 \le n \le 31$). $y[n]$ denotes the linear convolution of $x_1[n]$ and $x_2[n]$

$$y[n] = x_1[n] * x_2[n].$$

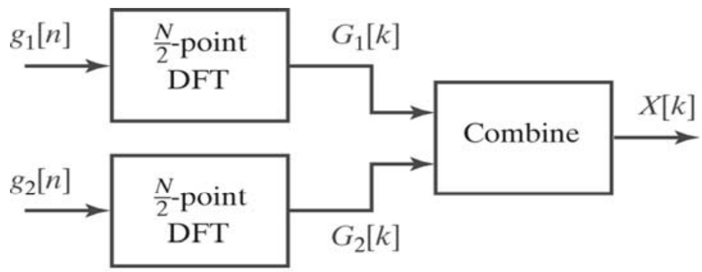


(a) Determine all values of N for which all the values of $y[n]$ can be completely recovered from $x_5[n]$.

(b) Specify explicitly how to recover $y[n]$ from $x_5[n]$ for the **smallest** value of N which you determined in part a). (Hint: Think carefully how long $x_3[n]$ and $x_4[n]$ are.)

Question 2. (20 points) FFT

The system in the figure below computes an N-point (where N is an even number) DFT $X[k]$ of an N-point sequence $x[n]$ by decomposing $x[n]$ into two $N/2$-point sequences $g_1[n]$ and $g_2[n]$, computing the $N/2$-point DFT's $G_1[k]$ and $G_2[k]$, and then combining these to form $X[k]$.



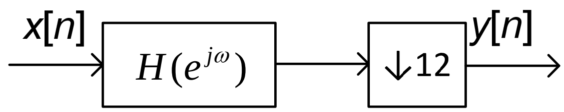
(a) If $g_1[n]$ is the even-indexed values of $x[n]$ and $g_2[n]$ is the odd-indexed values of $x[n]$, that is, $g_1[n] = x[2n]$ and $g_2[n] = x[2n+1]$, then $X[k]$ will be the DFT of $x[n]$.

For $N = 4$, draw the butterfly diagram to show the combine operation. Is this algorithm decimation-in-time or decimation-in-frequency?

(b) In using the system in Figure 2, an error is made in forming $g_1[n]$ and $g_2[n]$, such that $g_1[n]$ is **incorrectly** chosen as the odd-indexed values and $g_2[n]$ as the even indexed values but $G_1[k]$ and $G_2[k]$ are still combined as in part a) and the incorrect sequence $\hat{X}[k]$ results. Express $\hat{X}[k]$ in terms of $X[k]$.

Question 3. (20 points) Filter design

Suppose we want to design a low pass filter $H(e^{j\omega})$ to be used in a filter bank with the following decimation structure:



The filter specification is as follows:

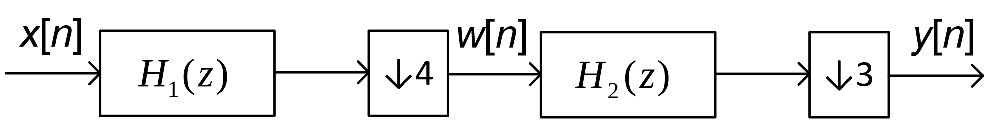
Sampling frequency:	96 kHz.
Passband:	$0 \le f \le 3900$ Hz.
Transition band:	3900 Hz $\le f \le$ 4000 Hz.
Stopband:	4000 Hz $\le f \le$ 48 kHz.
Passband ripple:	$\delta_1 = 0.01$
Stopband ripple:	$\delta_2 = 10^{-4}$.

1) Single-stage design
   a. Sketch the frequency response $H(e^{j\omega})$ (ignoring ripples), specifying all the passband and stopband frequency edges.
   b. Using Kaiser's formula, estimate the length of this filter (remember the length is M+1)

$$M = \frac{-10\log_{10}(\delta_1\delta_2) - 13}{2.324\Delta\omega}$$

   What type of filter is this?
   If this filter is designed using the *remez* algorithm, how many alternations can it have? (There may be more than one answer.)

2) Now we want to use a 2-stage design as follows to simplify the filters (so that the new filters have shorter lengths).



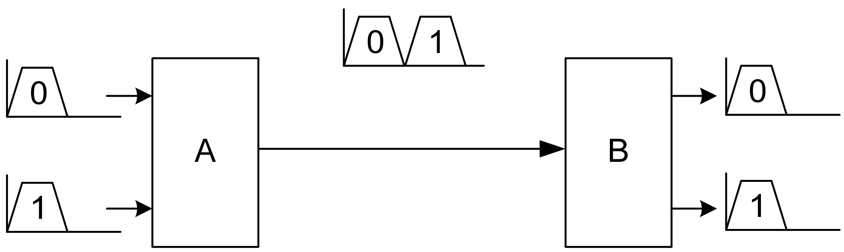
   a. What is the relation between $H(z)$ and $H_1(z)$, $H_2(z)$?
   What are the sampling frequencies at $w[n]$ and $y[n]$?

   b. Each of $H_1(z)$ and $H_2(z)$ is a lowpass filter. Sketch the frequency responses $H_1(e^{j\omega})$ and $H_2(e^{j\omega})$ with shortest filter length possible such that the effective system is the same as the single-stage one.

   Let the passband ripples of $H_1(e^{j\omega})$ and $H_2(e^{j\omega})$ be $\delta_1/2$ and the stopband ripples remain as δ_2. Estimate the lengths of these 2 new filters.

Question 4. (20 points) Multirate

In order to make efficient use of a communications channel, two separate frequency bands are used to transmit data. Digital frequencies $0 \le \omega \le \pi/2$ are used for band "0", and digital frequencies $\pi/2 \le \omega \le \pi$ are used for band "1". Our goal is to take two input channels, each with frequencies from $0 \le \omega \le \pi/2$ and combine them to form a single data stream, and then split them at the other end of the channel.



The only tools at our disposal are:

- Up-samplers (by a factor of 2)
- Down-samplers (by a factor of 2)
- Ideal low-pass filters, with cutoff frequency $\pi/2$.
- Ideal high-pass filters, with cutoff frequency $\pi/2$.
- Adders
- Multipliers (by some constant coefficient).

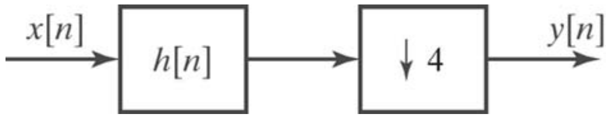
Note: Specifically, we do not have a modulator or demodulator, nor can we multiply two signals together.

(a) Draw the block diagram to implement the function of box "A", and draw the frequency response at each point in your diagram.

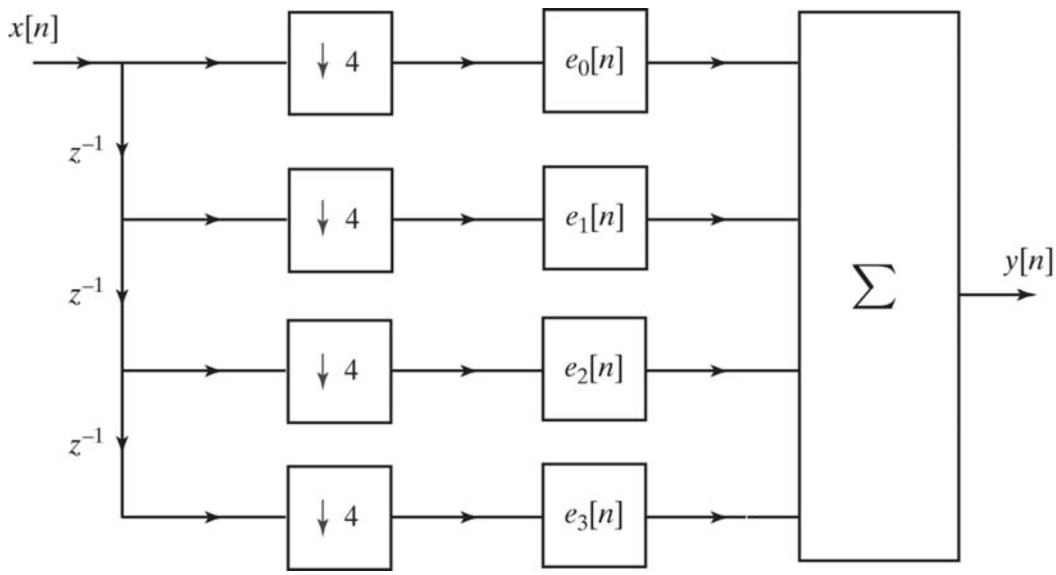
(b) Draw the block diagram to implement the function of box "B", and draw the frequency response at each point in your diagram.

Question 5. (20 points) Implementation

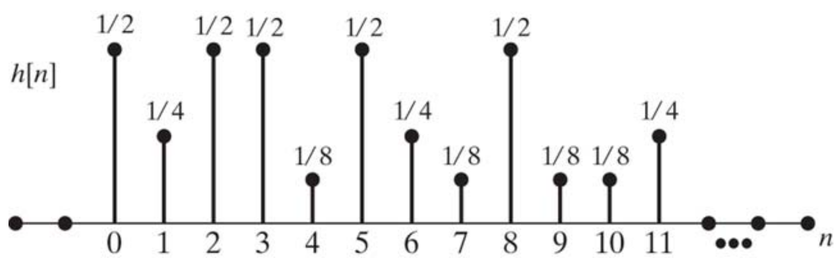
Consider the system shown in the figure below.



We want to implement this system using the polyphase structure as follows.



Assume $h[n]$ is defined as in the figure below ($h[n] = 0$ for all $n < 0$ and $n \geq 12$).



(a) Draw the sequences $e_0[n]$, $e_1[n]$, $e_2[n]$, and $e_3[n]$ that result in a correct implementation.

(b) For the polyphase implementation, determine the number of multiplies per output sample for the overall system. Also, determine the number of multiplies per input sample for the overall system.

With the choice of $e_0[n]$, $e_1[n]$, $e_2[n]$, and $e_3[n]$ from part (a), can you reduce the number of multiplies any further? Explain.

(c) Draw a direct form implementation of $e_0[n]$.

Question 6. (20 points) Conceptual Questions

Indicate whether the following statements are true or false. If true, give a brief explanation. If false, give a simple counter-example or a clear reason.

a) An all pass filter can be FIR.

b) Increasing the window length can lower stopband ripples.

c) All periodic continuous-time signals remain periodic after sampling.

d) For filters designed with the *remez* algorithm, the transition band can be non-monotonic.

e) Rate conversion by first upsampling by L then downsampling by M is the same as first downsampling by M then upsampling by L.

f) FFT algorithms always result in the number of computation of the order $N\log_2 N$.

g) In FFT decimation-in-frequency algorithms, the output frequency samples are in order.

h) Bilinear transformation of analog IIR filter to digital IIR filter can preserve linear phase.

i) A digital filter cannot have a monotonic passband.

j) A symmetric FIR filter with linear phase when implemented with finite-precision still remains linear-phase.

DSP Fact Sheet

Convolution: $$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

z-transform: $$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$$

Fourier Series: $$x(n) = \sum_{k=0}^{N-1} c_k W_N^{-kn}, \quad c_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n)W_N^{nk}, \quad \text{where } W_N = e^{-j2\pi/N}$$

Fourier Transform: $$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}, \quad x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)e^{j\omega n}d\omega$$

Identity: $$\frac{1}{N}\sum_{k=0}^{N-1} W_N^{nk} = \sum_{l=-\infty}^{\infty} \delta(n-lN) = \begin{cases} 1 & n = lN \\ 0 & \text{otherwise} \end{cases}$$

Table 1 z-transform Pairs

sequence	z-transform	ROC
$\delta(n)$	1	all z
$u(n)$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
$a^n u(n)$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$na^n u(n)$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
$-a^n u(-n-1)$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
$r^n \cos(\omega_o n)\, u(n)$	$\frac{1-r\cos\omega_o z^{-1}}{1-2r\cos\omega_o z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > r$
$r^n \sin(\omega_o n)\, u(n)$	$\frac{r\sin\omega_o z^{-1}}{1-2r\cos\omega_o z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > r$

Table 2 z-transform Properties

sequence	z-transform
$a^n x(n)$	$X(z/a)$
$x^*(n)$	$X^*(z^*)$
$x(-n)$	$X(1/z)$
$x(n) * h(n)$	$X(z)H(z)$
$x(n-k)$	$z^{-k}X(z)$