Homework 2
Due: October, 7, 2011

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5.28. A causal LTI system has the system function

$$H(z) = \frac{(1 - e^{j\pi/3}z^{-1})(1 - e^{-j\pi/3}z^{-1})(1 + 1.1765z^{-1})}{(1 - 0.9e^{j\pi/3}z^{-1})(1 - 0.9e^{-j\pi/3}z^{-1})(1 + 0.85z^{-1})}.$$

(a) Write the difference equation that is satisfied by the input $x[n]$ and output $y[n]$ of this system.

(b) Plot the pole–zero diagram and indicate the ROC for the system function.

(c) Make a carefully labeled sketch of $|H(e^{j\omega})|$. Use the pole–zero locations to explain why the frequency response looks as it does.

(d) State whether the following are true or false about the system:

(i) The system is stable.

(ii) The impulse response approaches a nonzero constant for large n.

(iii) Because the system function has a pole at angle $\pi/3$, the magnitude of the frequency response has a peak at approximately $\omega = \pi/3$.

(iv) The system is a minimum-phase system.

(v) The system has a causal and stable inverse.

5.33. $H(z)$ is the system function for a stable LTI system and is given by:

$$H(z) = \frac{(1 - 2z^{-1})(1 - 0.75z^{-1})}{z^{-1}(1 - 0.5z^{-1})}.$$

(a) $H(z)$ can be represented as a cascade of a minimum-phase system $H_{\text{min1}}(z)$ and a unity-gain all-pass system $H_{\text{ap}}(z)$, i.e.,

$$H(z) = H_{\text{min1}}(z)H_{\text{ap}}(z).$$

Determine a choice for $H_{\text{min1}}(z)$ and $H_{\text{ap}}(z)$ and specify whether or not they are unique up to a scale factor.

(b) $H(z)$ can be expressed as a cascade of a minimum-phase system $H_{\text{min2}}(z)$ and a generalized linear-phase FIR system $H_{\text{lp}}(z)$:

$$H(z) = H_{\text{min2}}(z)H_{\text{lp}}(z).$$

Determine a choice for $H_{\text{min2}}(z)$ and $H_{\text{lp}}(z)$ and specify whether or not these are unique up to a scale factor.

5.34. A discrete-time LTI system with input $x[n]$ and output $y[n]$ has the frequency response magnitude and group delay functions shown in Figure P5.34-1. The signal $x[n]$, also shown in Figure P5.34-1, is the sum of three narrowband pulses. In particular, Figure P5.34-1 contains the following plots:

- $x[n]$
- $|X(e^{j\omega})|$, the Fourier transform magnitude of a particular input $x[n]$
- Frequency response magnitude plot for the system
- Group delay plot for the system

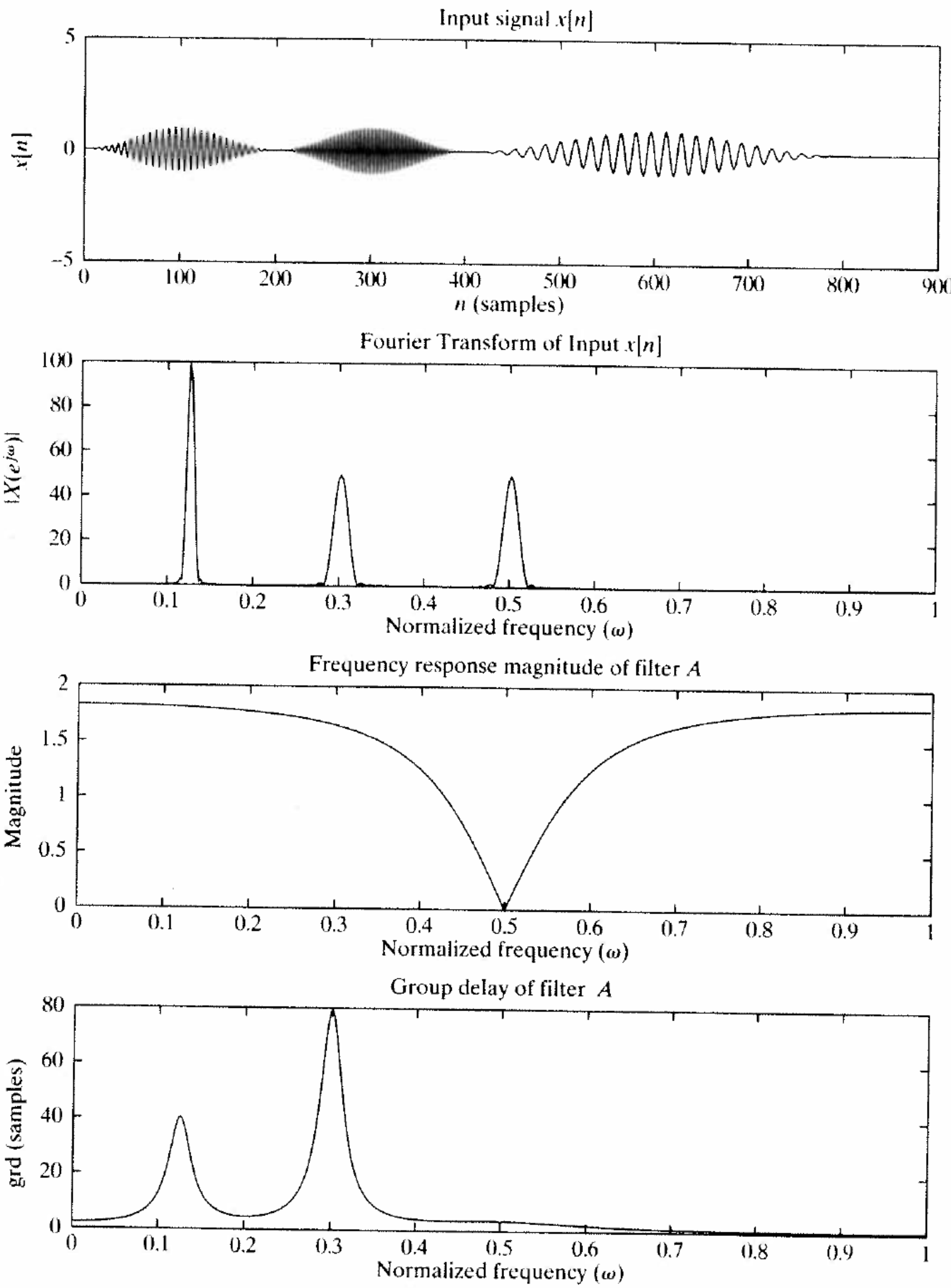

Figure P5.34-1 The input signal and the filter frequency response

In Figure P5.34-2 you are given four possible output signals, $y_i[n]$ $i = 1, 2, \ldots, 4$. Determine which one of the possible output signals is the output of the system when the input is $x[n]$. Provide a justification for your choice.

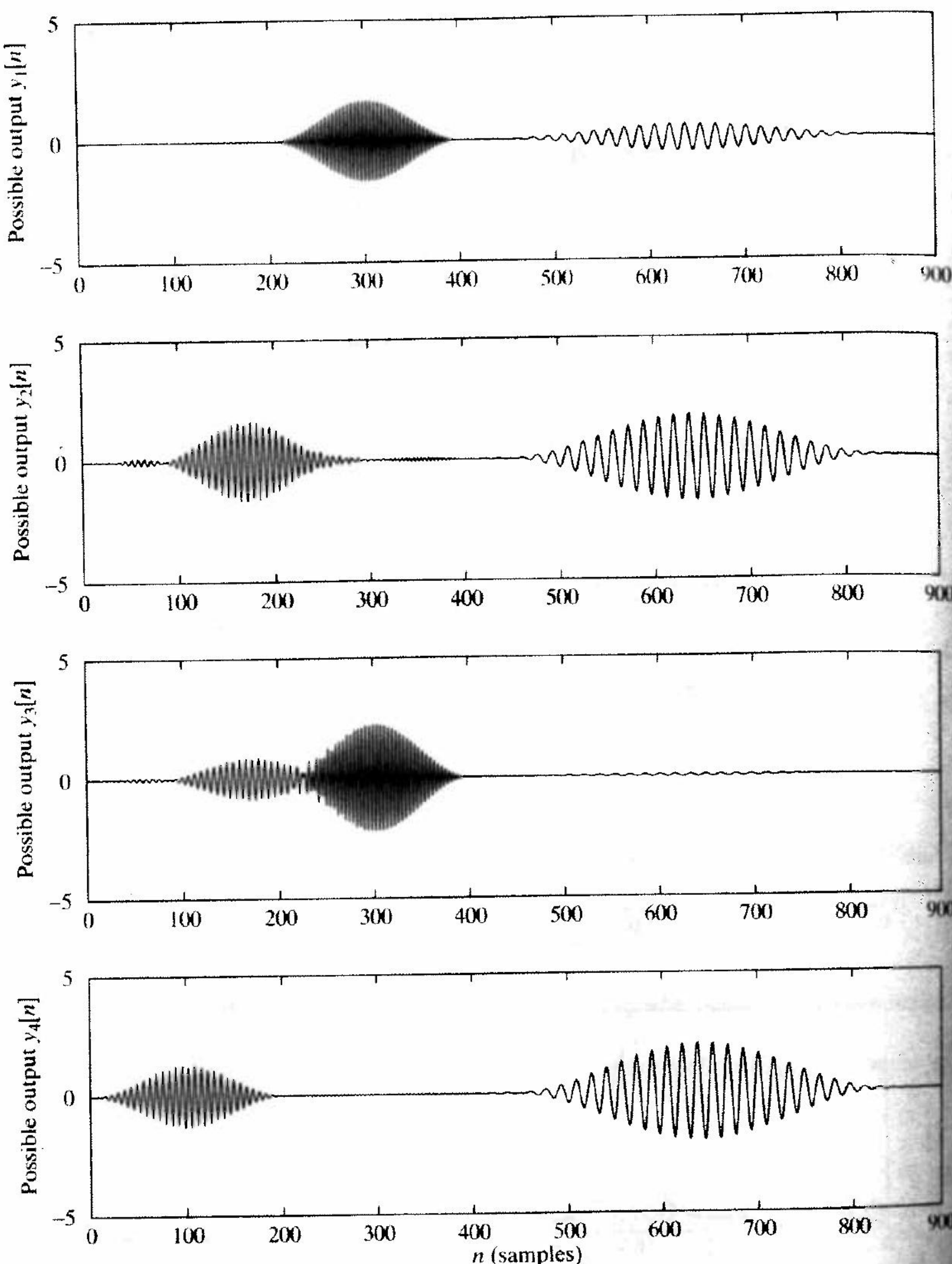

Figure P5.34-2 Possible output signals

5.45. The pole–zero plots in Figure P5.45 describe six different causal LTI systems.

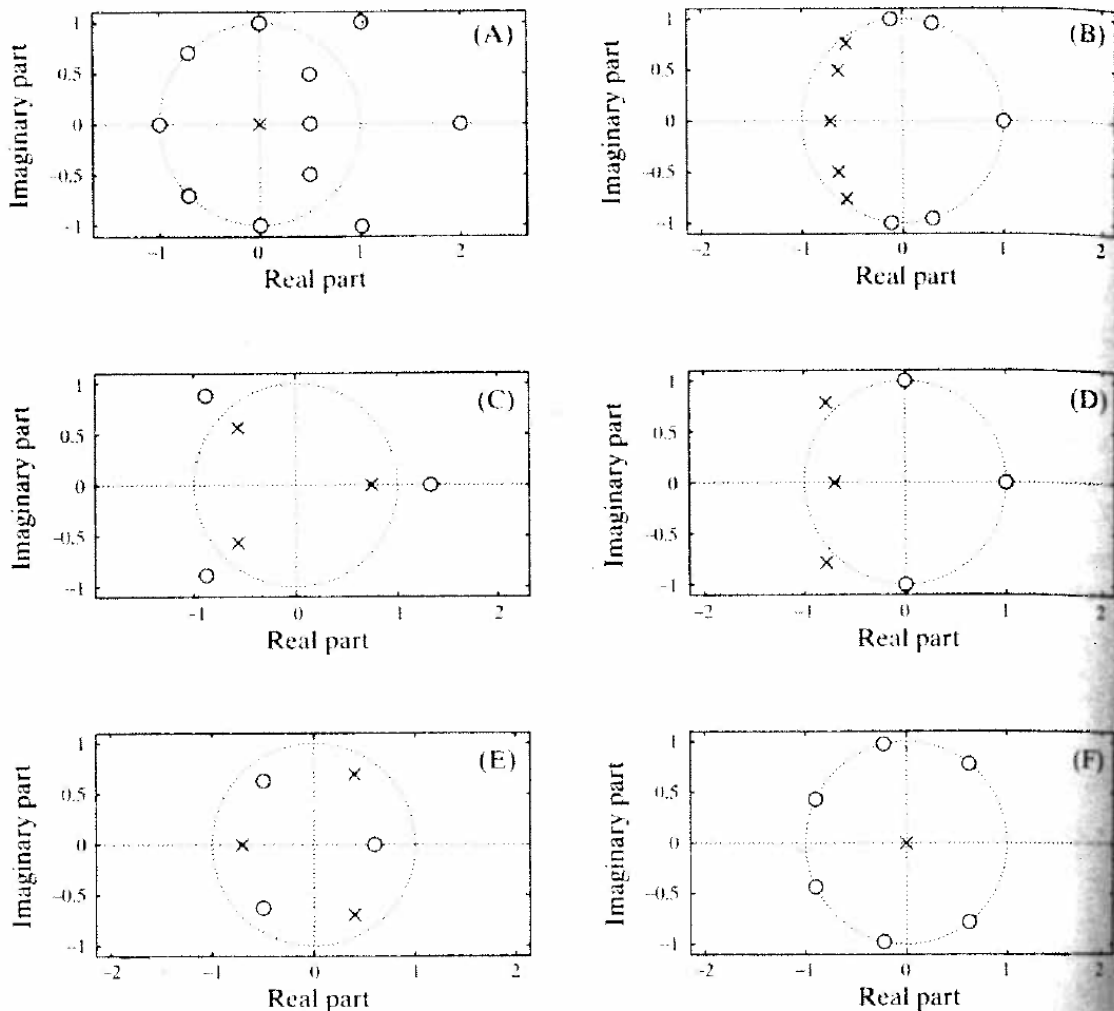

Figure P5.45

Answer the following questions about the systems having the above pole–zero plots. In each case, an acceptable answer could be *none* or *all*.

(a) Which systems are IIR systems?
(b) Which systems are FIR systems?
(c) Which systems are stable systems?
(d) Which systems are minimum-phase systems?
(e) Which systems are generalized linear-phase systems?
(f) Which systems have $|H(e^{j\omega})|$=constant for all ω?
(g) Which systems have corresponding stable and causal inverse systems?
(h) Which system has the shortest (least number of nonzero samples) impulse response?
(i) Which systems have lowpass frequency responses?
(j) Which systems have minimum group delay?

5.59. **(a)** A specific minimum-phase system has system function $H_{min}(z)$ such that

$$H_{min}(z)H_{ap}(z) = H_{lin}(z),$$

where $H_{ap}(z)$ is an all-pass system function and $H_{lin}(z)$ is a causal generalized linear-phase system. What does this information tell you about the poles and zeros of $H_{min}(z)$?

(b) A generalized linear-phase FIR system has an impulse response with real values and $h[n] = 0$ for $n < 0$ and for $n \geq 8$, and $h[n] = -h[7-n]$. The system function of this system has a zero at $z = 0.8e^{j\pi/4}$ and another zero at $z = -2$. What is $H(z)$?

5.70. It is not possible to obtain a causal and stable inverse system (a perfect compensator) for a nonminimum-phase system. In this problem, we study an approach to compensating for only the magnitude of the frequency response of a nonminimum-phase system.

Suppose that a stable nonminimum-phase LTI discrete-time system with a rational system function $H(z)$ is cascaded with a compensating system $H_c(z)$ as shown in Figure P5.70.

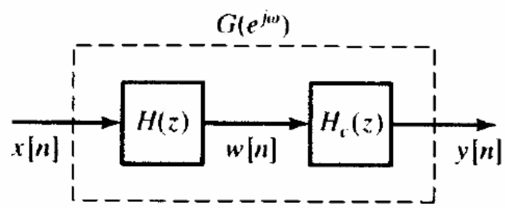

Figure P5.70

(a) How should $H_c(z)$ be chosen so that it is stable and causal and so that the magnitude of the overall effective frequency response is unity? (Recall that $H(z)$ can always be represented as $H(z) = H_{ap}(z)H_{min}(z)$.)

(b) What are the corresponding system functions $H_c(z)$ and $G(z)$?

(c) Assume that

$$H(z) = (1 - 0.8e^{j0.3\pi}z^{-1})(1 - 0.8e^{-j0.3\pi}z^{-1})(1 - 1.2e^{j0.7\pi}z^{-1})(1 - 1.2e^{-j0.7\pi}z^{-1}).$$

Determine $H_{min}(z)$, $H_{ap}(z)$, $H_c(z)$, and $G(z)$ for this case, and construct the pole–zero plots for each system function.

Q7: Matlab Question

■ 6.4 Phase Effects for Lowpass Filters

Filters are often designed to meet a specification of the magnitude of the frequency response. In this exercise, you will see that in some situations the effect of the phase can be important as well. You will examine the effect of processing clean speech and noisy speech with several filters which have roughly the same frequency response magnitude, and compare how the outputs of the different filters sound.

These problems require you to work with three different lowpass filters which are specified by coefficient vectors stored in the file `phdist.mat`, which is in the Computer Explorations Toolbox. If this file is already in your MATLABPATH, you can type `load phdist`. If this file has been loaded correctly, then typing `who` should result in

```
>> who
Your variables are:
a1        a3        b2        xnoise
a2        b1        b3
```

The vectors `a1` through `a3` and `b1` through `b3` contain the coefficient vectors for three discrete-time filters in the format required by `freqz` and `filter`. The vector `xnoise` contains a speech signal with some high-frequency noise added. You will also want to load the original version of this speech signal, which is the file `orig.mat` in the Computer Explorations Toolbox. If this file is in your MATLABPATH, you can load it by typing `load orig`. The uncorrupted version of this signal should now be in the vector `x`.

Basic Problems

(a). Use `freqz` to compute the frequency response of all three filters at 1024 evenly spaced points on the interval $0 \leq \omega < \pi$. Plot the frequency response magnitude of all three filters. Suppose you used the same speech signal as the input to all three filters. Based on the magnitudes of the frequency responses, which two of the output signals would you expect to sound the most similar?

(b). Generate appropriately labeled plots of the phase of the frequency response for all three systems. The function `angle` will only return angles between $-\pi$ and π. If you just plot the output of `angle`, you may find it hard to understand some of the plots, since the phase will "wrap-around" from the top to the bottom of the graph every time it passes π or $-\pi$. The function `unwrap` takes the output of `angle` and removes the "wrap-around" effect to obtain a more continuous phase function. For example, if `H` contains the samples of the frequency response at the frequencies specified in `omega`, then `plot(omega/pi,unwrap(angle(H)))` displays the unwrapped phase. Which two filters are the most alike in phase response?

(c). Define the vectors `y1` through `y3` to be the output of the filters when they are applied to the clean speech signal in `x` using `filter`. Play each speech signal using `soundsc`. Is the speech still easy to understand in each case? Which two speech signals sound most alike? Based on your results for this input signal, would you say that the magnitude or phase of these filters is more important?