Homework 3
Due: October, 21, 2011

Question 1. Textbook 4.23
Question 2. Textbook 4.31
Question 3. Textbook 4.39
Question 4. Textbook 4.44
Question 5. Textbook 4.47
Question 6. Sampling and Quatization
Question 7. Matlab Question

4.23. Figure P4.23-1 shows a continuous-time filter that is implemented using an LTI discrete-time filter ideal lowpass filter with frequency response over $-\pi \le \omega \le \pi$ as

$$H(e^{j\omega}) = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & \omega_c < |\omega| \le \pi. \end{cases}$$

(a) If the continuous-time Fourier transform of $x_c(t)$, namely $X_c(j\Omega)$, is as shown in Figure P4.23-2 and $\omega_c = \frac{\pi}{5}$, sketch and label $X(e^{j\omega})$, $Y(e^{j\omega})$ and $Y_c(j\Omega)$ for each of the following cases:

(i) $1/T_1 = 1/T_2 = 2 \times 10^4$
(ii) $1/T_1 = 4 \times 10^4$, $1/T_2 = 10^4$
(iii) $1/T_1 = 10^4$, $1/T_2 = 3 \times 10^4$.

(b) For $1/T_1 = 1/T_2 = 6 \times 10^3$, and for input signals $x_c(t)$ whose spectra are bandlimited to $|\Omega| < 2\pi \times 5 \times 10^3$ (but otherwise unconstrained), what is the maximum choice of the cutoff frequency ω_c of the filter $H(e^{j\omega})$ for which the overall system is LTI? For this maximum choice of ω_c, specify $H_c(j\Omega)$.

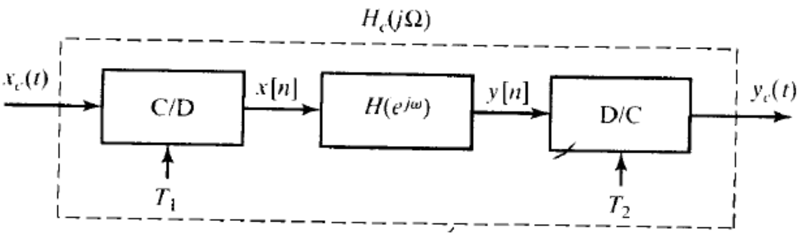


Figure P4.23-1

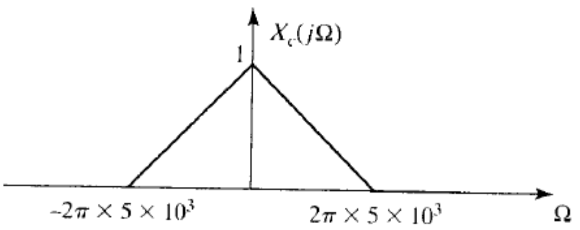


Figure P4.23-2

4.31. Figure P4.31-1 shows the overall system for filtering a continuous-time signal using a discrete-time filter. The frequency responses of the reconstruction filter $H_r(j\Omega)$ and the discrete-time filter $H(e^{j\omega})$ are shown in Figure P4.31-2.

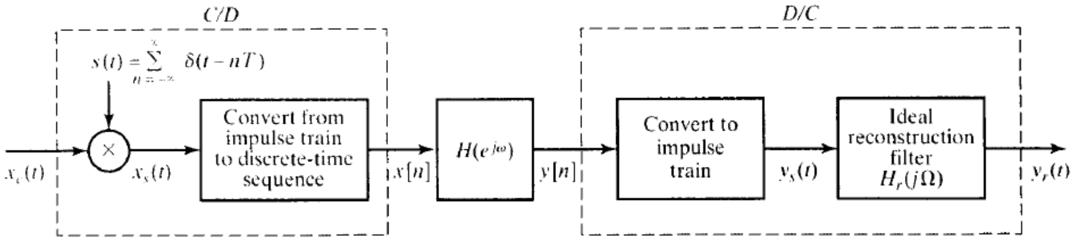


Figure P4.31-1

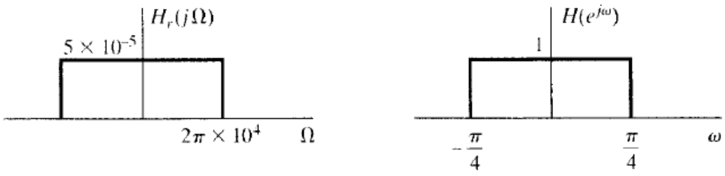


Figure P4.31-2

(a) For $X_c(j\Omega)$ as shown in Figure P4.31-3 and $1/T = 20$ kHz, sketch $X_s(j\Omega)$ and $X(e^{j\omega})$.

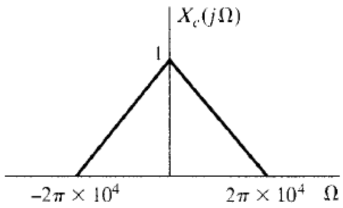


Figure P4.31-3

For a certain range of values of T, the overall system, with input $x_c(t)$ and output $y_c(t)$, is equivalent to a continuous-time lowpass filter with frequency response $H_{eff}(j\Omega)$ sketched in Figure P4.31-4.

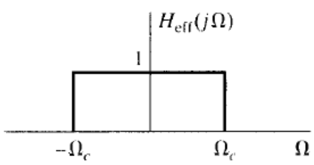


Figure P4.31-4

(b) Determine the range of values of T for which the information presented in (a) is true when $X_c(j\Omega)$ is bandlimited to $|\Omega| \leq 2\pi \times 10^4$ as shown in Figure P4.31-3.

(c) For the range of values determined in (b), sketch Ω_c as a function of $1/T$.

Note: This is one way of implementing a variable-cutoff continuous-time filter using fixed continuous-time and discrete-time filters and a variable sampling rate.

4.39. In system A, a continuous-time signal $x_c(t)$ is processed as indicated in Figure P4.39-1.

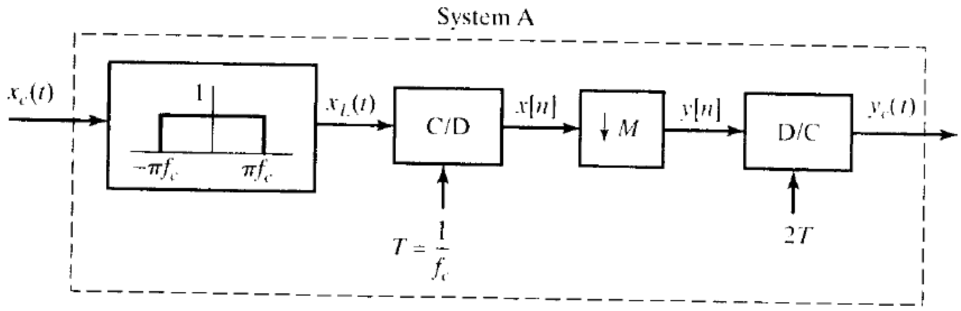


Figure P4.39-1

(a) If $M = 2$ and $x_c(t)$ has the Fourier transform shown in Figure P4.39-2, determine $y[n]$. Clearly show your work on this part.

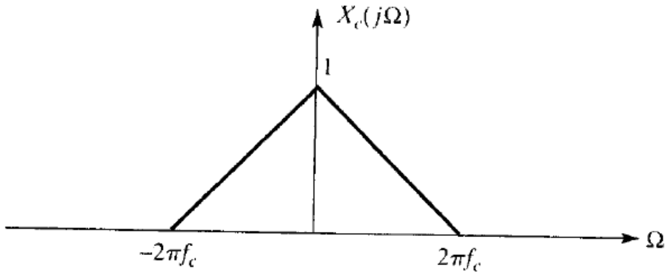


Figure P4.39-2

We would now like to modify system A by appropriately placing additional processing modules in the cascade chain of system A (i.e., blocks can be added at any point in the cascade chain—at the beginning, at the end, or even in between existing blocks). All of the current blocks in system A must be kept. We would like the modified system to be an ideal LTI lowpass filter, as indicated in Figure P4.39-3.

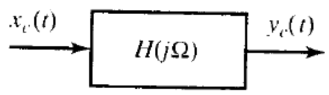


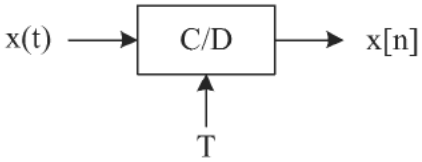
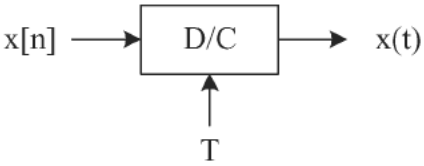
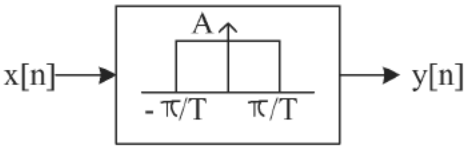
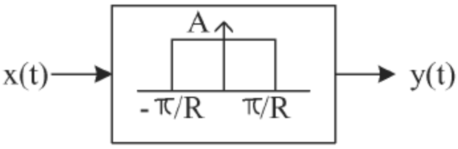
Figure P4.39-3

$$H(j\Omega) = \begin{cases} 1 & |\Omega| < \frac{2\pi f_c}{5} \\ 0 & \text{otherwise} \end{cases}$$

We have available an unlimited number of the six modules specified in the table given in Figure P4.39-4. The per unit cost for each module is indicated, and we would like the final cost to be as low as possible. **Note that the D/C converter is running at a rate of "$2T$".**

(b) Design the lowest-cost modified system if $M = 2$ in System A. Specify the parameters for all the modules used.

(c) Design the lowest-cost modified system if $M = 4$ in System A. Specify the parameters for all the modules used.

	**Continuous to Discrete Time Converter** Parameters: T Cost : 10
	**Discrete to Continuous Time Converter** Parameters: T Cost : 10
	**Discrete Time Low Pass Filter** Parameters: A, T Cost : 10
	**Continuous Time Low Pass Filter** Parameters: A, R Cost : 20
	**Expander** Parameters: L Cost : 5
	**Compressor** Parameters: M Cost : 5

P4.39-4

4.44. Consider the two systems shown in Figure P4.44.

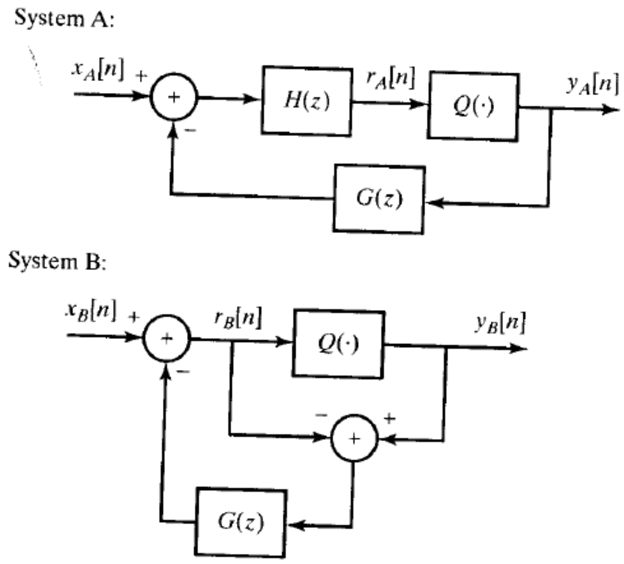


Figure P4.44

where $Q(\cdot)$ represents a quantizer which is the same in both systems. For any given $G(z)$, can $H(z)$ always be specified so that the two systems are equivalent (i.e., $y_A[n] = y_B[n]$ when $x_A[n] = x_B[n]$) for any arbitrary quantizer $Q(\cdot)$? If so, specify $H(z)$. If not, clearly explain your reasoning.

4.47. Consider the system shown in Figure P4.47-1 for discrete-time processing of the continuous-time input signal $g_c(t)$.

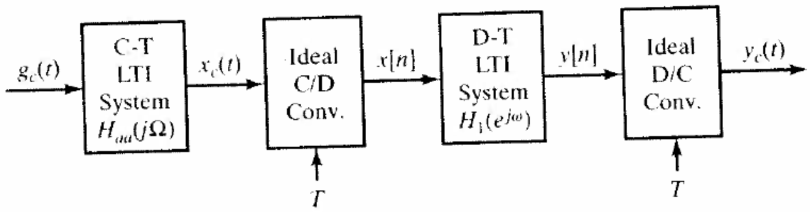


Figure P4.47-1

The continuous-time input signal to the overall system is of the form $g_c(t) = f_c(t) + e_c(t)$ where $f_c(t)$ is considered to be the "signal" component and $e_c(t)$ is considered to be an "additive noise" component. The Fourier transforms of $f_c(t)$ and $e_c(t)$ are as shown in Figure P4.47-2.

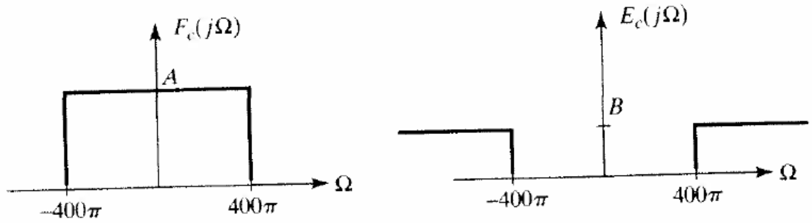


Figure P4.47-2

Since the total input signal $g_c(t)$ does not have a bandlimited Fourier transform, a zero-phase continuous-time antialiasing filter is used to combat aliasing distortion. Its frequency response is given in Figure P4.47-3.

$$H_{aa}(j\Omega) = \begin{cases} 1 - |\Omega|/(800\pi) & |\Omega| < 800\pi \\ 0 & |\Omega| > 800\pi \end{cases}$$

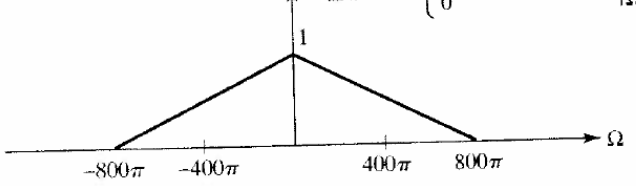


Figure P4.47-3

(a) If in Figure P4.47-1 the sampling rate is $2\pi/T = 1600\pi$, and the discrete-time system has frequency response

$$H_1(e^{j\omega}) = \begin{cases} 1 & |\omega| < \pi/2 \\ 0 & \pi/2 < |\omega| \le \pi \end{cases}$$

sketch the Fourier transform of the continuous-time output signal for the input whose Fourier transform is defined in Figure P4.47-2.

(b) If the sampling rate is $2\pi/T = 1600\pi$, determine the magnitude and phase of $H_1(e^{j\omega})$ (the frequency response of the discrete-time system) so that the output of the system in Figure P4.47-1 is $y_c(t) = f_c(t - 0.1)$. You may use any combination of equations or carefully labeled plots to express your answer.

(c) It turns out that since we are only interested in obtaining $f_c(t)$ at the output, we can use a lower sampling rate than $2\pi/T = 1600\pi$ while still using the antialiasing filter in Figure P4.47-3. Determine the minimum sampling rate that will avoid aliasing distortion of $F_c(j\Omega)$ and determine the frequency response of the filter $H_1(e^{j\omega})$ that can be used so that $y_c(t) = f_c(t)$ at the output of the system in Figure P4.47-1.

(d) Now consider the system shown in Figure P4.47-4, where $2\pi/T = 1600\pi$, and the input signal is defined in Figure P4.47-2 and the antialiasing filter is as shown in Figure P4.47-3.

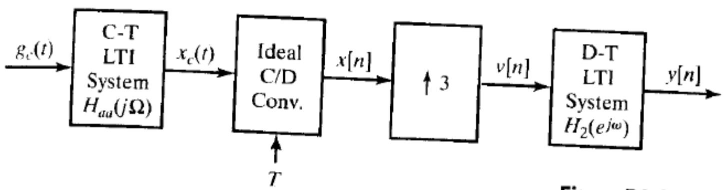


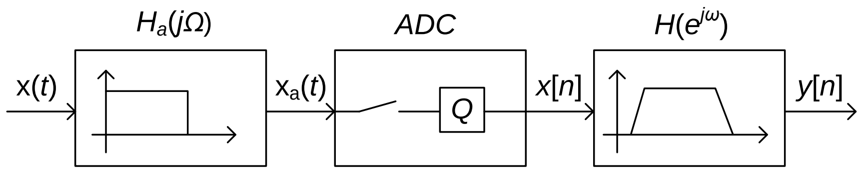
Figure P4.47-4 Another System Block Diagram

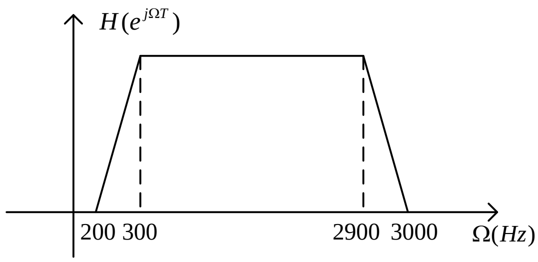
where

$$v[n] = \begin{cases} x[n/3] & n = 0, \pm 3, \pm 6, \ldots \\ 0 & \text{otherwise} \end{cases}$$

What should $H_2(e^{j\omega})$ be if it is desired that $y[n] = f_c(nT/3)$?

Problem 6. Sampling and quantization





a) In designing a telephone system, a near-ideal analog anti-aliasing filter $H_a(j\Omega)$ of bandwidth 4 KHz is used before the sampling circuit (such that the output of $H_a(j\Omega)$ has no energy at frequencies above 4 KHz).

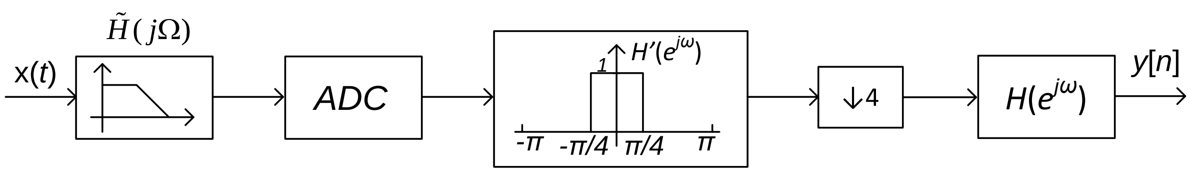
   The digital bandpass filter is designed to have the passband from 300 Hz to 2900Hz and stopband edges at 200 Hz and 3000 Hz as shown in the above figure.

   What is the minimum sampling frequency such that there is no aliasing in $y[n]$? (Note that there may be aliasing in $x[n]$ as long as $H(e^{jw})$ can remove it.) Sketch the frequency responses of $x_a(t)$, $x[n]$ and $y[n]$. Assume the frequency response of x_a is the same as $H_a(j\Omega)$.

b) If the sampling rate is 8 kHz and the ADC uses 4-sigma signal scaling and quantizes to a total of 10 bits per sample, what is the quantization SNR (in dB) at $x[n]$?

   If the SNR has to be greater than 50 dB, how many sigma (σ_x) will you need to fit in the maximum signal amplitude of the ADC? (Assume the same number of bits per sample.)

c) Now we want to use oversampling to relax the anti-aliasing filter as in the figure below. Suppose that the nominal sampling rate is 8 kHz but we choose to oversample by 4 times.



   Let

$$\tilde{H}_a(j\Omega) = \begin{cases} 1, & |\Omega| \le \Omega_p \\ 0, & |\Omega| > \Omega_s \end{cases}$$

   Find the smallest Ω_p and the largest Ω_s such that the above system is equivalent to the original system (at sampling rate 8 kHz), ignoring the effect on quantization noise.

d) With 4-sigma signal scaling and 10 bits per sample, what is the quantization SNR in the system with oversampling?

■ 7.5 Half-Sample Delay

The effect of multiplying a discrete-time Fourier transform (DTFT) $X(e^{j\omega})$ by an exponential $e^{-j\omega n_d}$ is to shift the signal $x[n]$ in the time-domain. If n_d is a positive integer, this is easily understood as delaying the discrete-time signal to obtain $x[n - n_d]$. When n_d is not an integer, the effect is more difficult to understand, since discrete-time signals are only defined for integer values of their arguments. In this exercise, you will explore the effect of a system whose frequency response is

$$H(e^{j\omega}) = e^{-j\omega/2}, \quad |\omega| < \pi. \tag{7.10}$$

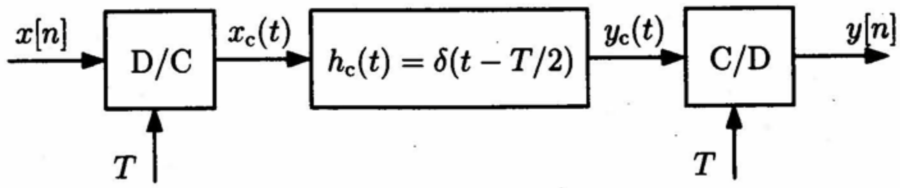


Figure 7.2. The half-sample delay of a discrete-time signal $x[n]$ can be constructed by first determining $x_c(t)$, the bandlimited interpolation of $x[n]$. The half-sample delay $y[n]$ is given by sampling $y_c(t)$, a delayed version of the continuous-time signal $x_c(t)$.

The output of this system for the input $x[n]$ is certainly not "$x[n-1/2]$", since discrete-time signals are not defined for noninteger values of their independent variable. In this exercise, you will see that the output of this system, known as a "half-sample delay," can be thought of as samples of a delayed version of the bandlimited interpolation of $x[n]$ as shown in Figure 7.2.

Basic Problems

(a). The impulse response of the half-sample delay system in Eq. (7.10) is

$$h[n] = \frac{\sin(\pi(n-1/2))}{\pi(n-1/2)}. \tag{7.11}$$

Define `h` to contain the values of $h[n]$ for the interval $-31 \le n \le 32$. Generate a plot of `h` using `stem`.

(b). For the system whose impulse response is stored in `h`, compute the frequency response at 1024 evenly spaced frequencies between 0 and π using `freqz`. Generate an appropriately labeled plot of the magnitude of the frequency response. Could you have predicted the value of $H(e^{j\omega})|_{\omega=\pi}$ based on your plot from Part (a)?

(c). For the half-sample delay system with the frequency response shown in Eq. (7.10), the group delay for the system is a constant n_d. Because the group delay is a constant, all frequencies will be delayed equally as they pass through the system. Use the function `grpdelay` to compute the group delay of the system with the impulse response contained in `h`. Exercise 6.5 explains the use of the `grpdelay` function. Is n_d an integer for this system? Note that `grpdelay` assumes the system is causal and that `h(1)` $= h[0]$, so that the values returned by `grpdelay` will be larger than expected from Eq. (7.11).

(d). Use the function `sinc` to define `x` to be the signal

$$x[n] = \frac{\sin(\pi n/8)}{\pi n/8} \tag{7.12}$$

for $-127 \le n \le 127$. Compute the output of the LTI system with impulse response `h` to this signal using `filter`, and store the result in `y`.

(e). Use **subplot** to generate plots of the input and output of the system. Note that the output signal has an initial transient. To make a better comparison, discard the first 31 samples of **y**. Note that **x** has a unique maximum at $n = 0$. Does **y** attain its maximum value at a unique sample? Is the output simply a delayed version of the input, or is the effect of the system on the input more complicated than that?

(f). Define **y2** to be the output of the system with impulse response **h** when you use **y** as the input. Effectively, this is cascading two half-sample delay systems together. Use **subplot** to compare **y2** with the original input **x**. Does **y2** have a unique maximum? How are these two signals related? Define the impulse response of the overall system **h2** to be the convolution of **h** with itself using **conv**. Plot **h2** using **stem**. Does this make sense given what you saw in your plot of **y2** and **x**?

Intermediate Problems

(g). Consider computing $x_c(t)$ as the bandlimited interpolation of $x[n]$ with $T = 1$, i.e., assume the values of $x_c(nT)$ are given by $x[n]$ and between these samples the continuous-time signal is obtained by interpolating with a continuous-time lowpass filter with a cutoff frequency of π. Write an analytic expression for $x_c(t)$ for the signal $x[n]$ defined in Part (d).

(h). The group delay you found in Part (c) had the units of samples. Converting to continuous time, we have $T = 1$ sec/sample. How many seconds should the continuous-time signal $x_c(t)$ be delayed to introduce a delay equivalent to that generated by the discrete-time system with impulse response given by **h**? Write an analytic expression for $y_c(t)$, the result of delaying the interpolated signal $x_c(t)$ by this amount.

(i). Now that you have defined $y_c(t)$ to be a delayed version of the bandlimited interpolation of the input, you need only resample this signal to compute the half-sample delay. Define **yc** to be the values obtained by sampling $y_c(t)$ every $T = 1$ seconds. Using the **hold** function, generate a plot where the values of **yc** are connected by solid lines using **plot**, while **y** is plotted using **stem**. Other than the initial transient, do the values of **y** fall on the line given by **yc**?